ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 3, Summer 1992

NO CONTINUUM IN E^2 HAS THE TMP;
II. TRIODIC CONTINUA

L.D. LOVELAND

ABSTRACT. A subset X of the Euclidean plane E^2 has the triple midset property (TMP) if, for every line segment $[x, y]$ in E^2 such that $\{x, y\} \subset X$, the perpendicular bisector of $[x, y]$ meets X in exactly three points. Resolving the planar aspect of a more general question, the main theorem shows that no compact, connected, nondegenerate subset of E^2 can possess this triple midset property.

1. Introduction. Let (X, ρ) be a metric space, and let x and y be two points of X. The *midset* $M(x, y)$ of x and y is the set of all points m of X such that $\rho(x, m) = \rho(y, m)$. If each of its midsets consists of two points, the metric space X is said to have the *double midset property* (DMP); for example, a circle in the Euclidean plane E^2 has the DMP. It has been conjectured that a continuum with the DMP must be homeomorphic to a simple closed curve, a conjecture which has been confirmed for continua lying in E^2 [**3**]. A metric space in which every midset consists of three points is said to have the *triple midset property* (TMP), but no example is available of a continuum with the TMP. In this paper I show that no such continuum exists in E^2, the space where one might first look for examples.

Although no examples have been found of continua with the TMP, it follows from a theorem of Bagemihl and Erdős [**1**] that there exists a subset of E^2 with the property that its intersection with every line consists of three points. Such a three-point set has the TMP. Mazurkiewicz [**6**] had previously demonstrated the existence of a subset E^2 that meets every line in exactly two points.

Midsets have also been called bisectors [**2**] or equidistant sets [**8, 9**], but, for subsets of Euclidean spaces, it is helpful to distinguish between

Received by the editors on November 25, 1989, and in revised form on February 2, 1990.
1991 AMS *Mathematics Subject Classification.* Primary 54F50, 54F15, Secondary 54F65, 51K05, 51M05.
Key words and phrases. Arc, bisector, continuum, Euclidean plane, equidistant set, midset, simple closed curve, triod, triple midset property.

Copyright ©1992 Rocky Mountain Mathematics Consortium