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THEORY OF THE GENERAL H-FUNCTION OF TWO VARIABLES

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ABSTRACT. Rather simple criteria are provided for the determination of the convergence regions of the double Mellin-Barnes integrals which is used to define the H-functions of two variables. Several specific examples are included.

1. Introduction. In 1978 the general H-function of two variables was defined [**2**] by the double Mellin-Barnes integral of the form

$$(1)\quad H[x,y;(\alpha,a,A)_n^m;L_s,L_t] = \frac{1}{(2\pi i)^2}\int_{L_t}\int_{L_s}\frac{\prod_{j=1}^m\Gamma(\alpha_j+a_js+A_jt)}{\prod_{j=m+1}^n\Gamma(\alpha_j+a_js+A_jt)}x^sy^t\,ds\,dt,$$

where $\alpha_j\in\mathbf{C}$; $a_j,A_j\in\mathbf{R}$, $j=1,2,\dots,n$; m,n are nonnegative integers and $m\le n$. Here L_s and L_t are infinite contours in the complex s- and t-planes, respectively, such that $\alpha_j+a_js+A_jt\ne 0,-1,-2,\dots$ for arbitrary $j=1,2,\dots,m$.

Two monographs [**6, 9**] and more than 250 papers are dedicated to the study of the H-functions (1) or somewhat specialized forms thereof. However, the works of many of these authors, for example [**1, 7, 8**], contain inaccuracies in finding the conditions on the parameters α_j,a_j,A_j that provide the convergence of the integral in (1). This fact was noticed and was made precise by Buschman [**2**].

In this paper rather simple criteria are provided for the determination of the convergence of integral (1) in terms of the parameters α_j,a_j,A_j, $j=1,2,\dots,n$.

2. Convergence regions results.

Theorem. *Let the contours L_s and L_t have vertical form, i.e.,* $\operatorname{Re}(s)$ *and* $\operatorname{Im}(s)$ *are restricted for $s\in L_s$, $t\in L_t$. Then the integral*

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