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AN EMBEDDING THEOREM AND ITS CONSEQUENCES

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ABSTRACT. It is well known that if X is a Tychonoff space and $F \subset C^*(X)$ separates points and closed sets then the evaluation map e_F corresponding to the family F is an embedding. When e_F is an embedding it does not necessarily follow that F separates points and closed sets. In this note we prove a general embedding theorem and then we use the theorem to characterize exactly those $F \subset C^*(X)$ for which e_F are embeddings. In fact, in our characterization, $C^*(X)$ may be replaced by $C(X)$.

Introduction: Let X be a Tychonoff space and $C^*(X)$ be the set of all real valued bounded continuous functions defined on X. In **1**] Ball and Yokura defined $\mathcal{E}(X)$ as the collection of all subsets F of $C^*(X)$ for which the evaluation maps e_F are embeddings. One of our main purposes is to determine the elements of $\mathcal{E}(X)$. If e_F is an embedding, then F generates the T_2-compactification $e_F X$ of X (see [**2**]). This reason leads us to characterize those $F \subset C^*(X)$ whose evaluation map e_F are embeddings. Some characterizations of the members of $\mathcal{E}(X)$ follow from the embedding theorems of Mrowka [**5**, Theorem 2.1] and Engelking [**4**, p. 122, Problem 2.3.D].

The notion "weakly separates points and closed sets" has been introduced for a family of functions from a topological space to each member of a family of topological spaces. We then use the notion to prove a general embedding theorem which is simpler than those of [**5**] and [**4**]. We use this theorem to characterize the elements of $\mathcal{E}(X)$. We conclude the paper by establishing a generalized version of Lemma 3.5 of [**1**].

1. In order to state and prove our embedding theorem we begin by recalling the following:

Let F be a family of functions $f : X \to Y_f$ where X and Y_f for all $f \in F$ are topological spaces. Then the evaluation map corresponding

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