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FREE-BY-FINITE CYCLIC AUTOMORPHISM GROUPS

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ABSTRACT. Motivated by an example of J.L. Dyer, we consider a group G whose automorphism group is isomorphic to the fundamental group of a graph of locally cyclic groups. The conclusion is that G is infinitely generated Abelian and $\operatorname{Aut} G$ is free-by-finite cyclic with all torsion elements having order dividing 8, 10, 12 or 30.

1. Introduction. It is a natural problem to attempt to characterize those groups which arise as the full automorphism group of some group, or, at least, to ask whether such groups share any common structural features. While little (if any) evidence exists to suggest any reasonably general answers to this question, some progress has been made (notably in [**3, 5, 12**]) on the less ambitious problem of finding large classes of groups whose structure precludes them from being automorphism groups. The purpose of this note is to record some further examples of this type. (For other work along these lines see, for example, [**4, 7, 10, 11, 14**].)

The result described here is combinatorial in flavor and evolved from an attempt to understand in a more general context an observation of J.L. Dyer [**4**] that $SL_2(\mathbf{Z})$ is not the automorphism group of any finitely generated group. Dyer's demonstration depends on the well-known presentation of $SL_2(\mathbf{Z})$ as an amalgam $\mathbf{Z}_4 *_{\mathbf{Z}_2} \mathbf{Z}_6$. A fact which makes this example of particular interest is that, while $\mathbf{Z}_2 * \mathbf{Z}_2$ is the only freely decomposable automorphism group [**5**], among amalgams in general, automorphism groups appear to be not at all uncommon [**9**]. Thus, it is reasonable to ask whether Dyer's example is an isolated one or whether it is representative of a class of examples. The gist of the following result is that Dyer's observation applies to the fundamental group of any graph of locally cyclic groups.

Theorem. *Let A be a treed* HNN *group with locally cyclic vertex*

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