

A CLASS OF STARLIKE FUNCTIONS

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ABSTRACT. We study the class of functions f which are analytic and univalent in the unit disk U , which map U onto a starlike domain and are normalized by $f(0) = 1$ and $f(p) = 0$, $0 < p < 1$. We obtain sharp bounds on the integral means of f and its derivatives centered at zero and p . These lead to sharp bounds on $|f^{(n)}(z)|$ for $n = 0, 1, 2, \dots$.

1. Introduction. The class of functions which are meromorphic and univalent in $U = \{z : |z| < 1\}$ with a simple pole at $z = p$, $0 < p < 1$, and which map U onto the complement of a starlike domain has been studied in a series of papers [1, 5, 6 and 7]. The reciprocals of these functions are a subclass of weakly starlike 1-valent functions, which were studied by Hummel [3, 4] in a more general setting. The functions in this subclass have the property that they map U onto a starlike domain and are normalized by $f(0) = 1$ and $f(p) = 0$. (Hummel did not require $f(0) = 1$). We will consider several extremal problems in this class. We obtain sharp bounds on the integral means of a function and its derivatives and also sharp bounds on the coefficients of the power series expansions about $z = 0$ and $z = p$.

2. The class $S^*(p)$. We denote by $S^*(p)$ the class of functions f which are analytic and univalent in U with $f(0) = 1$ and $f(p) = 0$, $0 < p < 1$, and which map U onto a starlike domain. (A starlike domain will always mean a domain starlike with respect to the origin.)

Theorem 1. *A function f with $f(0) = 1$, $f(p) = 0$, $0 < p < 1$, is in $S^*(p)$ if and only if for z in U ,*

$$(2.1) \quad \operatorname{Re} \left[\frac{(z-p)(1-pz)f'(z)}{f(z)} \right] > 0.$$

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