ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 1, Winter 1993

SPECIAL VALUE SET POLYNOMIALS OVER FINITE FIELDS

JAVIER GOMEZ-CALDERON

ABSTRACT. Let F_q denote the finite field of order q where q is a prime power. In this paper we prove that if m and n are two integers dividing $q-1$, $2 \leq m$, $2 \leq n$ and $d = mn < \sqrt[4]{q}$, then

$$\begin{aligned}\frac{2q}{2m+2n-1} &\leq |\{(x^m+b)^n : x \in F_q\}| \\ &\leq \min\{(q-1)/m, (q-1)/n\} + 1\end{aligned}$$

for all $0 \neq b$ in F_q.

1. Introduction. Let F_q denote the finite field of order q where q is a prime power. If $f(x)$ is a polynomial of degree d over F_q, let $V_f = \{f(x) : x \in F_q\}$ denote the image or value set of $f(x)$ and let $|V_f|$ denote the cardinality of V_f. It is clear that if f is of degree d,

$$[(q-1)/d] + 1 \leq |V_f| \tag{1}$$

where $[x]$ denotes the greatest integer $\leq x$. Hence,

$$[(q-1)/d] + 1 \leq |V_f| \leq q. \tag{2}$$

A permutation polynomial over F_q has a value set of maximal possible cardinality so that if $f(x)$ permutes F_q, then $|V_f| = q$. Many papers have been written concerning permutation polynomials over finite fields, with an excellent survey being given in Lidl and Niederreiter [**6**, Chapter 7] and Lidl and Mullen [**5**].

At the other extreme, a polynomial for which equality is achieved in (1) is called a minimal value set polynomial. Minimal value set polynomials over finite fields have been studied in Carlitz, Lewis, Miller and Straus [**1**] and Mills [**7**]. Recently, in [**4**], Gomez-Calderon and

Received by the editors on June 30, 1990, and in revised form on January 20, 1991.