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HALL SUBGROUPS OF ORDER NOT DIVISIBLE BY 3

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1. Introduction. In [**9**] it is proved that if a finite group G has a Hall π-subgroup and if π does not contain 2, then all Hall π-subgroups of G are conjugate. The proof of this is based upon proving it in the special case when G is a simple group. Further, it is shown that if H is a Hall π-subgroup of a finite simple group and $2 \notin \pi$, then H has a Sylow tower. An examination of the proof of this shows that a crucial point in the argument is that a Weyl group has very few Hall subgroups other than the Sylow subgroups. Indeed, if H is a Hall subgroup of a Weyl group and the order of H is divisible by at least two distinct primes, then H must have even order; one use of the assumption that $2 \notin \pi$ in the result quoted above is to assert that, when G is a simple group of Chevalley type, a Hall π-subgroup of the Weyl group of G must be a Sylow subgroup. Since if H is as above, it is also true that $|H|$ is divisible by 3 as well as by 2, it is tempting to consider Hall π-subgroups of a finite simple group where now 2 may belong to π but we exclude 3. The main result of this paper deals with the case when the simple group is either of the groups $A_n(q)$ or $C_n(q)$; specifically, we prove the following:

Proposition. *Let S be either $A_n(q)$ or $C_n(q)$. Assume S has a Hall π-subgroup with $3 \notin \pi$. Then all Hall π-subgroups of S are conjugate in S and a Hall π-subgroup of S has a Sylow tower. Further, if $S \le G \le \mathrm{Aut}\,(S)$, then G has a Hall π-subgroup, all Hall π-subgroups of G are conjugate in G, and a Hall π-subgroup of G is solvable.*

It should be noted that the results in this paper make use of the fact (an immediate consequence of the classification of finite simple groups) that the Suzuki groups are the only non-Abelian finite simple groups of order not divisible by 3. Using this, the above proposition is straightforward to prove when S is $A_n(q)$. When S is the Symplectic group $C_n(q)$, however, the argument is more difficult and also requires

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