ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 2, Spring 1993

HYPERGEOMETRIC FUNCTIONS OVER FINITE FIELDS AND REPRESENTATIONS OF SL $(2, q)$

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ABSTRACT. It is well known that the matrix elements in the principal representations of SL $(2, R)$ with respect to the appropriate basis are essentially hypergeometric functions. A parallel theory is presented here for the principal representations of SL $(2, F)$ where F is a finite field.

1. Introduction. There has been much work recently on special functions over finite fields. Evans [**4**] derived analogues for various extensions of beta integrals over finite fields. Helversen-Pasotto [**10**] derived Barnes integral analogues. Koblitz [**11**] introduced analogues of hypergeometric functions. An extensive study of many finite field analogues of orthogonal polynomials was conducted by Evans [**6**], and a similar study of hypergeometric functions was conducted by the author [**8, 9**].

That this work might be related to representation theory is indicated by the papers of Helversen-Pasotto [**10**] and Li and Soto-Andrade [**12**] where results follow from properties of representations of GL $(2, q)$ and GL $(3, q)$.

In this paper we demonstrate that, as in the classical case (see, for example, [**16**, Chapter VII]), hypergeometric functions over finite fields arise as matrix elements of certain representations of SL (2).

The organization of this paper is as follows. The construction of the principal series representations for SL $(2, q)$ as representation operators is given in Section 2. In Section 3, matrix elements of these representations are given with respect to a canonical basis. These matrix elements are described in terms of hypergeometric functions in Section 4 which also contains a description of how properties of hypergeometric functions are derived from this framework. Finally, comments, questions and related representations are given in Section 5.

Received by the editors on May 22, 1989.
Partially supported by NSF Grant DMS 8801131.