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NON-HOMEOMORPHIC DISJOINT SPACES WHOSE UNION IS ω^*

W.W. COMFORT AND AKIO KATO

ABSTRACT. For certain pairs $\langle \alpha, \beta \rangle$ of cardinals we show that the Stone-Čech remainder $\omega^* = \beta(\omega)\backslash\omega$ can be written in the form $\omega^* = \cup_{\xi<\alpha} C_\xi$ where the spaces C_ξ are pairwise disjoint, pairwise non-homeomorphic, countably compact, and dense in ω^*, with each $|C_\xi| = \beta$. In specific cases the condition that the spaces $\{C_\xi : \xi < \alpha\}$ are non-homeomorphic may be strengthened, as follows:

(i) $\alpha = 2^{\mathbf{c}}$, $c \leq \beta = \beta^\omega < 2^{\mathbf{c}}$: for $\xi < \alpha$ there is no one-to-one continuous function from C_ξ into $\cup_{\eta<\xi} C_\eta$.

(ii) $\omega \leq \alpha \leq 2^{\mathbf{c}}$, $\beta = 2^{\mathbf{c}}$: for $\eta < \xi < \alpha$ there is no continuous function from C_η onto C_ξ.

(iii) $1 \leq \alpha \leq 2^{\mathbf{c}}$, $\beta = 2^{\mathbf{c}}$: for $\xi < \alpha$ there is no one-to-one continuous function from C_ξ into $\omega^*\backslash C_\xi$.

1. Preliminaries. The symbol ω denotes the least infinite cardinal number and the countably infinite discrete topological space, and ω^* is the Stone-Čech remainder $\beta(\omega)\backslash\omega$. We consider only Tychonoff spaces, and we write $X \approx Y$ if X and Y are homeomorphic. The expression $X \subseteq_h Y$ means that X embeds into Y, i.e., there is $X' \subseteq Y$ such that $X \approx X'$.

For spaces X, Y and K with K compact and continuous $f : X \to Y \subseteq K$, the symbol $\bar{f}$ denotes the continuous function $\bar{f} : \beta X \to K$ such that $f \subseteq \bar{f}$. In this context we will consider repeatedly the question whether or not a point $p \in \beta X\backslash X$ satisfies $\bar{f}(p) \in Y$. We note in this connection that the choice of the enveloping compact space K is irrelevant. That is, if K and L are compact spaces containing Y and if f is continuous from X into Y, then for each $p \in \beta X$ the function $f_K = f : X \to Y \subseteq K$ satisfies $\bar{f}_K(p) \in Y$ if and only if the function $f_L = f : X \to Y \subseteq L$ satisfies $\bar{f}_L(p) \in Y$.

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