

MODIFICATIONS OF TOEPLITZ MATRICES: JUMP FUNCTIONS

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ABSTRACT. In [1] the authors have studied formal properties of the orthogonal polynomial sequences related to a modification of a measure on a Jordan curve by a finite number of jump functions in points of the curve. In this paper we analyze, for infinite Toeplitz matrices with nonsingular principal minors, the addition of jump functions in points placed outside (inside) the unit circle. Necessary and sufficient conditions for the regularity of the new Toeplitz moment matrix are provided and the relation with the polynomial modifications of the moments is established in a similar way to the Hankel case [8].

Introduction. Let μ be a Borel, finite and positive measure on the unit circle. In recent papers [1, 2, 3] we have studied formal properties of the monic orthogonal polynomial sequences (M.O.P.S.) corresponding to the measure u obtained by adding to μ a finite number of masses of Dirac on points of the circle. This problem appears in [6] and the generalization studied in [1] relates this type of modification with the polynomial ones.

In this paper we consider a situation as general as possible: Let M be an infinite, Hermitian and Toeplitz matrix with nonsingular principal minors. We consider a modification that represents an extension of the Dirac's delta in a point placed not necessarily on the circle. In Section 1 necessary and sufficient conditions for the new moment matrix \mathcal{M} to be Hermitian, Toeplitz and with nonsingular principal minors are given. In Section 2 the connection between this type of modification and others introduced in [5] are provided. Some interesting examples are presented in Section 3 and, finally, if the moment functional L associated to M is semiclassical in the sense of [9], then it is shown in Section 4 that \mathcal{L} (moment functional associated to \mathcal{M}) is semiclassical too. In this sense we extend a known result for Hankel matrices [8].

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