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LIMITS OF WEIGHTED SPLINES BASED ON PIECEWISE CONSTANT WEIGHT FUNCTIONS

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Weighted splines using piecewise constant weight functions as introduced by Salkauskas [**7**] have proven useful in the interpolation of rapidly varying data by C^1 piecewise cubics and have been successfully exploited by, for example, Foley [**2, 3, 4**], both in combination with Nielson's ν-splines [**6**] and in a bivariate analogue of tensor product interpolation.

In [**8**], the authors have shown that for any weight function which is piecewise constant on *some* partition (not necessarily that determined by the interpolation points), there exists an interpolant which minimizes the weighted semi-norm. As it turns out, it is also a piecewise cubic. We show here that for a broad class of weight functions there exist unique optimal interpolants which can be represented as uniform limits of piecewise cubic interpolants based on piecewise constant weight functions.

The existence proof is similar to the approach taken by Meinguet [**5**] in the construction of optimal multivariate interpolants in a semi-Hilbert space.

Theorem 1. *Suppose that we are given a data-set,* $(x_1, f_1), \dots, (x_N, f_N)$ *with* $x_1 < \cdots < x_N$. *Let* $w(x)$ *be a positive locally integrable weight function such that*

$$0 < m \le w(x) \le M < \infty \quad \text{on } [x_1, x_N],$$

$$w(x) = 1, \quad \forall\, x \notin [x_1, x_N],$$

and

$$1/w(x) \in L_1[x_1, x_N].$$

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