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REMARKS ON FUNCTIONS WITH POSITIVE REAL PART ON THE BALL IN C^n

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Let P denote the convex set consisting of the holomorphic functions on the unit ball B of C^n $(n > 1)$ which have positive real part and take the value 1 at 0. In a recent paper McDonald [**4**] extended some of the earlier results of Forelli [**1, 2, 3**] which described the known extreme points of P. The purpose of this paper is to elaborate upon one of McDonald's main results, Theorem 3b given below. Let us first review some of the results of Forelli and McDonald which led to this theorem.

Given an n-tuple $\varphi = (\varphi_1, \dots, \varphi_n)$ of nonnegative integers, choose $c_\varphi > 0$ so that the monomial $h_\varphi(z) = c_\varphi z^\varphi$ satisfies $||h_\varphi|| \equiv \sup\{|h_\varphi(z)| : z \in B\} = 1$. Forelli had shown

Theorem 1. *Let $\varphi_j > 0$ for $j = 1, 2, \dots, n$. The function $(1 + h_\varphi)/(1 - h_\varphi)$ is an extreme point of P if and only if $gcd(\varphi_j) = 1$.*

McDonald observed that, when $n > 1$, the functions h_φ are extreme points of the unit ball A in $H^\infty(B)$ if and only if $\varphi_j > 0$ for $j = 1, 2, \dots, n$. This led to

Theorem 2. *The extreme points in P are images, under the mapping $f \to (1 + f)/(1 - f)$ of extreme points in A which satisfy $f(0) = 0$.*

Remark 1. We observe from the above that, if $gcd(\varphi_j) = k > 1$ and if $\theta = \varphi/k$, then both h_θ and $h_\varphi = (h_\theta)^k$ are extreme in A, but only the image of h_θ is extreme in P.

More generally, if f is an extreme point of A, and if $m > 1$ is a positive integer, then [**5**, Corollary 3.2] f^m is extreme in A, but its image $(1 + f^m)/(1 - f^m)$ is not an extreme point of P [**2**].

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