ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 3, Summer 1993

SOME INTERESTING BANACH SPACES

R.C. JAMES

ABSTRACT. If a Banach space X has an unconditionally basic skipped-blocking finite-dimensional decomposition (UBSBFDD), then each of the properties RNP, KMP, PCP, and CPCP is equivalent to X not having a subspace isomorphic with c_0. If X fails PCP and is a subspace of a space with a UBSBFDD, then X has a subspace isomorphic with c_0. Many examples are described that have no c_0-subspaces and fail RNP. Some of these have PCP. Some fail PCP and have CPCP. Some are not contained in a space with an unconditional basis, but this remains an open question for others. Sufficient conditions are given for a boundedly complete skipped-blocking decomposition to imply CPCP.

Introduction. A bounded closed convex subset K of a Banach space X has the *Radon-Nikodým property* (RNP) if, for any finite-measure space (S, Σ, μ) and any μ-continuous measure $\lambda : \Sigma \to X$ with $\lambda(E)/\mu(E) \in K$ for each $E \in \Sigma$, there is a Bochner-integrable function $f : S \to X$ such that $\lambda(E) = \int_E f\, d\mu$ for each $E \in \Sigma$. For X to have RNP means that the unit ball has RNP.

A bounded closed convex subset K of a Banach space X has the *Krein-Milman property* (KMP) if each closed convex subset of K is the closure of the convex span of its extreme points. For X to have KMP means that the unit ball has KMP. A Banach space has KMP if it has RNP [**18**], but whether the converse is true remains an important unsolved problem.

A bounded closed convex subset K of a Banach space X has the *point-of-continuity property* (PCP) if, for each nonempty closed subset C of K, there is a point x of C such that the weak and norm topologies (restricted to C) coincide at x; K has the *convex-point-of-continuity property* (CPCP) if this condition is satisfied for all nonempty closed convex subsets of K. For X to have PCP (CPCP) means that the unit ball has PCP (CPCP). The CPCP was introduced in [**3**] to aid in

Received by the editors on October 25, 1990.
AMS *Subject Classification.* 46B20.
Key words. Convex set, point of continuity, skipped blocking, unconditional basis.