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BIFURCATION OF SYNCHRONIZED PERIODIC SOLUTIONS IN SYSTEMS OF COUPLED OSCILLATORS I: PERTURBATION RESULTS FOR WEAK AND STRONG COUPLING

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ABSTRACT. This paper concerns a class of differential equations that govern the evolution of indirectly coupled oscillators. We establish the existence of synchronized periodic solutions for weak and strong coupling under certain conditions. The stability of the periodic solutions is also analyzed.

1. Introduction. It is shown in [**14**] that a number of problems in physics, chemistry and biology lead to systems of ordinary differential equations that represent oscillatory subunits coupled indirectly through a passive medium. In this paper we study the case where the oscillators, which govern the states of the uncoupled subunits, are all identical. That is, we study the following system of ordinary differential equations.

$$\begin{aligned}\frac{dx_i}{dt} &= f(x_i) + \delta P(x_0 - x_i), \qquad i = 1, \dots, N,\\ \frac{dx_0}{dt} &= \varepsilon\delta P\bigg(\frac{1}{N}\sum_{i=1}^{N} x_i - x_0\bigg).\end{aligned} \tag{1}$$

Here the variable x_0 represents the state of the coupling medium through which the subunits are coupled. P is an $n \times n$ constant matrix of permeability coefficients or conductances, and the parameters ε^{-1} and δ measure the relative capacity of the coupling medium and the coupling strength, respectively [**8, 9**]. In the absence of coupling the evolution in the i^{th} subunit is governed by the n-dimensional system $dx_i/dt = f(x_i)$ and it is assumed that this system has a nonconstant periodic solution. We show when (1) has periodic solutions and analyze the stability of these periodic solutions.

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