ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 23, Number 4, Fall 1993

ALGEBRAIC VECTOR BUNDLES ON THE 2-SPHERE

RICHARD G. SWAN

Barge and Ojanguren [**1**] have recently shown that there is a 1–1 correspondence between algebraic and topological vector bundles on the 2-sphere. This raises the problem of whether it is possible to give a purely algebraic classification of these bundles. I will give here an affirmative answer to this question. In particular, this gives a new algebraic proof that the tangent bundle of S^2 is nontrivial. An algebraic proof of this was previously given by Kong [**7**]. The present proof is considerably simpler but applies only to the 2-sphere whereas Kong's method applies to all even dimensional spheres.

Ideally, one would expect such an algebraic proof to apply to all real closed ground fields without the need to appeal to the Tarski principle (cf. [**8**]). While the present proof is purely algebraic, it does not meet this criterion since (in Section 2) it makes use of the fact that the additive and multiplicative groups of the real numbers have Archimedean orderings. I do not know if there is any easy way to avoid this difficulty.

I have included a number of remarks pointing out connections with topological results. These are not essential for the algebraic results presented here. I have also included an exposition of the theory of symplectic modules in an appendix for the convenience of those readers not familiar with this theory.

I do not know if there is a quaternionic analog of these results. See [**14**] for more information on this case.

I would like to thank Hongnian Li for pointing out a number of misprints in an earlier version of this paper. I would also like to thank the referee for suggesting a number of improvements in the exposition.

1. Known results. If R is a commutative ring we let $P_n(R)$ be the set of isomorphism classes of finitely generated projective modules of

Received by the editors on November 29, 1991, and in revised form on May 20, 1992.
Partially supported by the NSF.

Copyright ©1993 Rocky Mountain Mathematics Consortium