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CONGRUENCE NETWORKS FOR STRONG SEMILATTICES OF REGULAR SIMPLE SEMIGROUPS

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1. Introduction and summary. Normal cryptogroups (or normal bands of groups) form the class of semigroups which are strong semilattices of completely simple semigroups. We consider here the more general class of semigroups which are strong semilattices of regular simple semigroups. We denote the latter by $S = [Y; S_\alpha, \varphi_{\alpha,\beta}]$ where Y is a semilattice, for each $\alpha \in Y$, S_α is a regular simple semigroup, and for $\alpha \geq \beta$, $\varphi_{\alpha,b} : S_\alpha \rightarrow S_\beta$ is a homomorphism. These homomorphisms satisfy the usual conditions and determine the multiplication of S. This is the semigroup on whose lattice of congruences $\mathcal{C}(S)$ we consider certain operators.

A congruence ρ on S can be expressed by means of a congruence aggregate $(\xi; \rho_\alpha)$ where $\xi \in \mathcal{C}(Y)$ and $\rho_\alpha \in \mathcal{C}(S_\alpha)$ are congruences satisfying certain conditions, and we write $\rho \sim (\xi; \rho_\alpha)$. We call $\operatorname{gl} \rho = \xi$ and $\operatorname{loc} \rho = (\rho_\alpha)$ the global and the local of ρ. These induce the global relation $\mathcal{G}$ and the local relation $\mathcal{L}$ on $\mathcal{C}(S)$ by

$$\lambda \mathcal{G} \rho \iff \operatorname{gl} \lambda = \operatorname{gl} \rho, \qquad \lambda \mathcal{L} \rho \iff \operatorname{loc} \lambda = \operatorname{loc} \rho.$$

Our "global and local operators" are induced by the greatest and the least elements of the equivalence classes of $\mathcal{G}$ and $\mathcal{L}$ as follows:

ρG and ρg are the greatest and the least elements $\mathcal{G}$-related to ρ, respectively,

ρL and ρl are the greatest and the least elements $\mathcal{L}$-related to ρ, respectively.

These produce the four operators G, g, L and l on $\mathcal{C}(S)$. We are interested in the semigroup generated by $A = \{G, g, L, l\}$. This semigroup will be represented by generators and relations.

As for general regular semigroups, we define $E(S)$ to be the set of idempotents of S,

$$\ker \rho = \{a \in S \mid a \rho e \text{ for some } e \in E(S)\}$$

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