

## FUZZY ALGEBRAIC VARIETIES

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**ABSTRACT.** The concept of a fuzzy algebraic variety is introduced in order to bring the current knowledge of fuzzy commutative ring theory to bear on the solution of nonlinear systems of equations of fuzzy singletons. It is shown for every finite-valued fuzzy ideal  $A$  of a polynomial ring in several indeterminates over a field with  $A(0) = 1$  that the fuzzy algebraic variety of  $A$  can be expressed as a union of fuzzy irreducible algebraic varieties, no one of which is contained in the union of the others.

**Introduction.** Rosenfeld's application [12] of the pioneering work of Zadeh [15] inspired the fuzzification of various algebraic structures. Liu [2] and Mukherjee and Sen [10] presented some of the earliest work on the fuzzification of an ideal of a ring. Since then the notions of fuzzy prime ideal, fuzzy primary ideal, the radical of a fuzzy ideal, and the fuzzy primary representation of a fuzzy ideal have been introduced and examined [1,3,4,5,6,7,10,11,13,14,16,17,18,19]. There are several natural ways to define these concepts, many of which have appeared in the literature. In the interesting work of Zadehi [16] and Kumbhojkar and Bapat [1], the various types of fuzzy prime ideals, fuzzy primary ideals, and the radical of a fuzzy ideal have been compared. In [7] various types of radicals of fuzzy ideals and fuzzy primary representations of fuzzy ideals have been compared in order to prepare the way for the study of nonlinear systems of equations of fuzzy singletons.

Up to this point, the fuzzification of concepts and results of commutative ring theory have had no apparent application. The purpose of this paper is to give some meaning to fuzzy commutative ring theory developed to this point and to put some direction to its further study. We bring fuzzy commutative ring theory to bear on a natural application area, namely, the solution of nonlinear systems of equations of fuzzy singletons. Let  $R$  denote the polynomial ring  $F[x_1, \dots, x_n]$  where  $F$  is a field and  $x_1, \dots, x_n$  are algebraically independent indeterminates over

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Received by the editor on September 9, 1991.

*Key words.* Fuzzy ideal, fuzzy algebraic variety, fuzzy primary representation, irreducible, fuzzy nonlinear system of equations.

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