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COMPATIBILITY EQUATIONS FOR ISOMETRIC EMBEDDINGS OF RIEMANNIAN MANIFOLDS

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0. Introduction. Let (M, g) be an n-dimensional Riemannian manifold with the Riemannian metric g. A C^1 mapping F of M into a Euclidean space $\mathbf{R}^{n+p}$ is a local isometric embedding if and only if F satisfies

$$\sum_{\alpha=1}^{n+p} \frac{\partial u^\alpha}{\partial x^i} \frac{\partial u^\alpha}{\partial x^j} = g_{ij}(x), \qquad 1 \leq i, j \leq n, \tag{7}$$

where $(x^1, \ldots, x^n)$ is a local coordinate system of M and $g_{ij}(x) = g(\partial/\partial x^i, \partial/\partial x^j)$. Since $g_{ij} = g_{ji}$, the number of equations in (7) is $n(n+1)/2$ and thus the system (7) is underdetermined if $p > n(n-1)/2$ and overdetermined if $p < n(n-1)/2$.

In this paper we study a method of prolongation of (7) and conditions on g_{ij} under which (7) can be prolonged to an elliptic system, and discuss some of their geometric consequences. We restrict our interest to the case $p \leq n(n-1)/2$, which is a necessary condition for an isometric embedding to be elliptic.

A *compatibility equation* of (7) is an equation obtained by prolongation, that is, a process of differentiation and algebraic operations on (7).

In Section 1 of this paper, we construct compatibility equations of (7) by a method due to A. Finzi [**5**] and show that the classical equations of Gauss are compatibility equations of this type. These equations, which will be called compatibility equations of Finzi type, are the consequences of the cancellation of the principal parts in the process of prolongations of the original system. Thus they reveal properties of the solutions that are not exposed in the principal part.

In Section 2 we prove that a hypersurface H of M is characteristic for a certain system of compatibility equations if and only if H is an

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