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ASYMPTOTIC STABILITY AND THE DERIVATIVES OF SOLUTIONS OF FUNCTIONAL DIFFERENTIAL EQUATIONS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. We consider asymptotic stability of the zero solution of the functional differential equation $X'(t) = F(t, X_t)$ by Liapunov's second method with a basic condition

$$V'_{(1)}(t, X_t) \leq -\eta_1(t)W_1(m(X_t)) - \eta_2(t)W_2\left(\int_{t-h}^{t} |X(s)||X'(s)|\, ds\right),$$

or a similar condition. Some examples are given. As a consequence, the condition that $F(t,\phi)$ is bounded if ϕ is bounded is weakened in a classical result of stability of Krasovskii.

1. Introduction. The objective of this paper is to investigate asymptotic stability of the zero solution of the functional differential equation

$$X'(t) = F(t, X_t), \tag{1}$$

where $X_t(\theta) = X(t + \theta)$ for $-h \leq \theta \leq 0$ and h is a positive constant. Before proceeding we shall set forth some notation and terminology that will be used throughout this paper. Denote by C the space of continuous functions $\phi : [-h, 0] \to R^n$. For $\phi \in C$ we will use the norm $||\phi|| := \max_{-h \leq s \leq 0} |\phi(s)|$, where $|\cdot|$ is any convenient norm in R^n. Given $H > 0$, C_H denotes the set of $\phi \in C$ with $||\phi|| < H$. $X'(t)$ denotes the right-hand derivative at t if it exists and is finite. It is supposed that $F : R_+ \times C_H \to R^n$, that F is continuous, and that it

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