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COMPETITION MODELS IN THE GRADOSTAT WITH GENERAL NUTRIENT UPTAKE FUNCTIONS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. Mathematical models of two competing microbial species in the gradostat have been studied extensively in recent years. Much analysis of the model is based on the assumption that the nutrient uptake function is the Michaelis-Menten term in the Monod growth model. This paper shows that previous results, especially for uniform persistence and coexistence, can also be applied to models assuming more general nutrient uptake functions.

1. Introduction. In this paper we study mathematical models of two species competing for a single growth-limiting nutrient in the gradostat assuming general nutrient uptake functions.

The gradostat is a laboratory device simulating an ecosystem with nonuniform spatial distribution of nutrient supply. The original system was used by Wimpenny and Lovitt to study the effects of nutrient gradients on microbial growth [**10, 22, 23**]. Mathematical models of microbial growth and competition in the gradostat have been studied extensively [**8, 9, 17, 18, 19**]. While conditions for uniform persistence of the two species have been established, coexistence equilibrium as a global attractor of trajectories of the model equations has only been established for a very special case of a two-vessel gradostat using the Michaelis-Menten nutrient uptake function proposed by Monod [**12**].

The Michaelis-Menten nutrient uptake function is a prototype of monotonically increasing, bounded, and concave functions. Even though the Monod model is a popular model for microbial growth, other functions with similar properties are better approximations to some experimental data [**1**, Figure 2]. It is also possible that the function may not even be concave. From a mathematical point of view, it is desirable to study a model which can describe a wider class of nutrient

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