

**EXISTENCE AND UNIQUENESS OF GLOBAL  
SOLUTIONS FOR A SIZE-STRUCTURED MODEL  
OF AN INSECT POPULATION WITH  
VARIABLE INSTAR DURATION**

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Dedicated to Paul Waltman on the occasion of his 60th birthday

In [12], R.M. Nisbet and W.S.C. Gurney show how to construct mathematical models of the population dynamics of an insect which has an arbitrary number of instars (life stages) the duration of each of which is dependent on the insect achieving a threshold weight gain. Based on the general theory of size (mass)-structured population dynamics, see, e.g., [11], they obtain a system of delay differential equations of threshold-type for the number (density) of individuals in each instar as a function of time. By a threshold-type delay we mean a delay  $\tau = \tau_i(t)$ , which is determined by a threshold condition

$$(0.1) \quad \int_{t-\tau_i}^t g_i(s) ds = m_2 - m_1$$

where the rate of increase of weight

$$\frac{dm}{dt} = g_i(t)$$

is given by a prescribed function or by a dynamical variable  $g_i(t)$ . The meaning of (0.1) is that an individual emerging (graduating) from the  $i$ 'th instar at time  $t$  must have spent time  $\tau_i = \tau_i(t)$  in the  $i$ 'th instar, the instar being characterized by insect weight belonging to the interval  $(m_1, m_2)$ . The rate of weight gain  $g_i(t)$  may, for example, be given by

$$g_i(t) = G_i(F(t))$$

where  $F(t)$  is the food density which might also be included in the system as a dynamically changing variable.

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