ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 1, Winter 1994

# PLANTING AND HARVESTING FOR PIONEER-CLIMAX MODELS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. Kolmogorov-type systems of ordinary differential equations are presented, where per capita growth rates are either monotone decreasing (pioneer) or one-humped (climax) functions of weighted population densities. Varying an intraspecific crowding parameter destabilizes an equilibrium via Hopf bifurcation. This effect may be reversed by planting the pioneer population or harvesting the climax population. Averaging methods are used to study the two-dimensional system with constant rate or periodic rate planting.

**1. Introduction.** Competition and cooperation among different individuals and different species in an ecosystem for its natural resources are important factors in determining the development of the ecosystem. For example, a tree in a forest competes with its neighbors for light, space, carbon dioxide, and soil nutrients. Although the intensity of this competition may or may not be affected by the species type of the neighboring trees, it is affected by neighboring population density. Analogously, an animal may not care what type of competitor is consuming its food, but the amount of food consumed will be affected by competitor population density and, possibly, by species characteristics of the competitors, for instance, physical size. We try to model the effects of population density on the survival and growth of an individual species by assuming that the species' per capita growth rate (i.e., fitness) is a function of a weighted total density variable. This total density variable is a linear combination of the densities of the interacting species with coefficients weighting the intensity of the effect of each species. An example of such a model is the Lotka-Volterra system where the per capita growth rate is just a linear combination of the

Received by the editors on March 9, 1993.
Research supported by NSF grant DMS-9103829 and by the USDA Forest Service, Southeastern Forest Experiment Station, Pioneering (Population Genetics of Forest Trees) Research Unit, Raleigh, NC.