ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 1, Winter 1994

OSCILLATION AND ATTRACTIVITY IN A DIFFERENTIAL EQUATION WITH PIECEWISE CONSTANT ARGUMENTS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. Let $[\cdot]$ denote the greatest integer function. Consider the equation with piecewise constant arguments

$$(*) \qquad N'(t) = rN(t)\left(1-\sum_{j=0}^{m}(a_j N([t-k_j]) + b_j N^2([t-k_j]))\right)$$

where r is a positive number and for $j \in \{0, 1, \dots, m\}$, k_j is a nonnegative integer and a_j and b_j are nonnegative real numbers. We obtain necessary and sufficient conditions for all positive solutions of $(*)$ to oscillate about its positive equilibrium, we provide sufficient conditions for the positive equilibrium of $(*)$ to be a global attractor of all positive solutions, and we establish that every positive solution of $(*)$ is bounded from above and from below by positive constants.

1. Introduction and preliminaries. Let $[\cdot]$ denote the greatest integer function and let $\mathbf{R}$ denote the set of real numbers. Throughout this paper, unless otherwise specified, we will assume that $r \in (0, \infty)$ and for each $j \in \{0, 1, \dots, m\}$, k_j is a nonnegative integer and a_j and b_j are nonnegative real numbers with $a_j + b_j > 0$. When $k_j = 0$ for all j, we also assume $r \neq 1$.

In this paper we establish necessary and sufficient conditions for the oscillation of all positive solutions of the equation with piecewise constant arguments

$$(1) \qquad N'(t) = rN(t)\left(1-\sum_{j=0}^{m}(a_j N([t-k_j]) + b_j N^2([t-k_j]))\right)$$

about its positive equilibrium. We also obtain sufficient conditions for the positive equilibrium to be a global attractor of all positive solutions.

Received by the editors on March 9, 1993.
AMS 1980 *Subject Classification.* Primary 39A12, Secondary 39A10.