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A THEOREM OF MILLOUX FOR DIFFERENCE EQUATIONS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

We consider the vector difference equation

$$y(t+1) = A(t)y(t) \tag{1a}$$

where $A(t)$ is a complex $n \times n$ matrix function defined on the integer interval $[t_0, \infty) \equiv \{t_0, t_0 + 1, t_0 + 2, \dots\}$. We also will consider the difference operator analog of equation (1a)

$$\Delta y(t) = B(t)y(t) \tag{1b}$$

where $B(t)$ is a complex $n \times n$ matrix function defined on the integer interval $[t_0, \infty)$. An asterisk will denote the conjugate transpose and Δ will denote the forward difference operator, that is, $\Delta y(t) = y(t+1) - y(t)$.

The main theorem will give necessary and sufficient conditions for the existence of a nontrivial solution of equation (1a) which tends to zero. The theorem is a generalization of the trivial scalar case that all solutions of $u(t+1) = a(t)u(t)$ tend to zero provided the limit of $\prod a(t)$ is zero. The related result using equation (1b) is a discrete analog of a theorem of Hartman [**1, 2**] which gave necessary and sufficient conditions for the existence of a solution for a first order system of linear differential equations which tends to zero. Hartman's result implied a result of Milloux [**4**] for a certain second order scalar differential equation which demonstrated the existence of a nontrivial zero-tending solution.

Theorem 1A. *Assume*

$$\lim_{t\to\infty} ||y(t)||$$

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