

SUMMABILITY METHODS FOR OSCILLATION OF LINEAR SECOND-ORDER MATRIX DIFFERENTIAL EQUATIONS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. Familiar oscillation criteria of Wintner [11] and Hartman [7] for the equation (1) $y'' + q(t)y = 0$ on $[0, \infty)$ using limits of the mean $(1/t) \int_0^t (\int_0^s q(u) du) ds$ have been extended by various authors in many ways. For equation (1) itself, these extensions have included the use of weighted means [3, 10, 9] or of iterated weighted means [5]. Other extensions of the scalar results have been made to the matrix equation (2) $Y'' + Q(t)Y = 0$ [1, 2, 6] and to the self-adjoint matrix equation (3) $(PY')' + Q(t)Y = 0$ [4]. Meanwhile, Hartman [8] gave conditions on very general means which allowed simplification of the proofs in [5] and which properly included their results and some others. In this paper, all these considerations are combined to derive improved results for (5).

1. Introduction. In 1949, Wintner [11] proved that a hypothesis sufficient for the oscillation of

$$(1) \quad y'' + q(t)y = 0$$

is

$$(2) \quad \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t \int_0^s q(u) du ds = +\infty.$$

In 1952, Hartman [7] weakened this hypothesis to the following:

$$(3) \quad \begin{aligned} &\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t \int_0^s q(u) du ds > -\infty \quad \text{and} \\ &\lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t \int_0^s q(u) du ds \quad \text{does not exist.} \end{aligned}$$

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