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CYCLIC OPERATORS ON SHIFT COINVARIANT SUBSPACES

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ABSTRACT. Let ψ be a Blaschke product on the unit disk and denote by P_ψ the orthogonal projection of H^2 onto $H^2\theta\psi H^2$. Necessary and sufficient conditions for the adjoint $\{P_\psi T_\phi P_\psi\}^*$ of the compression of an analytic Toeplitz operator $T_\phi : F \to \phi F$ to $H^2\theta\psi H^2$ to be cyclic are given.

1. Introduction. Let H^2 and H^∞ denote the standard Hardy spaces on the unit disk $\mathbf{D} \equiv \{z \in \mathbf{C} : |z| < 1\}$. The standard unilateral shift S on H^2 is given by $S : F(z) \to zF(z)$.

For each function ϕ in H^∞, the analytic Toeplitz operator T_ϕ with symbol ϕ is a bounded linear operator on H^2 defined by $T_\phi : F \to \phi F$. The commutant of the shift operator S on H^2 is precisely the algebra $\{T_\phi : \phi \text{ is in } H^\infty\}$ of analytic Toeplitz operators. In [**8**], Wogen showed that there exists a fixed function in H^2 which is a cyclic vector for the adjoint T_ϕ^* of every analytic Toeplitz operator T_ϕ having nonconstant symbol ϕ (see Wogen [**8**, Theorem 1, p. 163]).

Let ψ be an inner function on the unit disk and denote by P_ψ the orthogonal projection of H^2 onto $H^2\theta\psi H^2$. Let ϕ be any function in H^∞. The compression $P_\psi T_\phi P_\psi$ of the analytic Toeplitz operator T_ϕ to the shift coinvariant subspace $H^2\theta\psi H^2$ is given by

$$P_\psi T_\phi P_\psi : F \to P_\psi(\phi F). \tag{1}$$

For $\phi(z) = z$, the operator $S_\psi \equiv P_\psi T_\phi P_\psi$ is the compression of the shift operator S to $H^2\theta\psi H^2$. Sarason has shown that a bounded linear operator T on $H^2\theta\psi H^2$ commutes with S_ψ if and only if T assumes the form (1) for some function ϕ in H^∞ (see Sarason [**5**, Theorem 1, p. 179]). Nikolskii points out that it would be of interest to determine those functions ϕ in H^∞ for which $P_\psi T_\phi P_\psi$ and $\{P_\psi T_\phi P_\psi\}^*$ are cyclic (see [**4**]). Incidental results have been obtained for the case

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