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STRONGLY REGULAR FUSIONS OF TENSOR PRODUCTS OF STRONGLY REGULAR GRAPHS

A.D. SANKEY

1. Introduction. A strongly regular graph (srg) is a regular graph, neither complete nor null, with two distinct vertices x and y having λ or μ common neighbors according to whether x and y are adjacent or not. (This and other definitions in this section will be repeated below.) An srg is an association scheme with three relations. We write f_0 for the identity relation, f_1 for adjacency, and f_2 for nonadjacency. By the tensor product, or Kronecker product, of two srg's with relations f_i and g_i, respectively, we mean the association scheme whose relations are $f_i \otimes g_j$, $0 \leq i, j \leq 2$. A *fusion* of the tensor product is a subscheme in the sense of Bannai [**1**]. We say a fusion is strongly regular if it is a rank 3 association scheme (hence its relations are those of an srg). A general problem for coherent configurations (cc's) is to decide under what circumstances the tensor product has coherent fusions. This work addresses that question for the specific case of srg's, which are equivalent to homogeneous rank 3 cc's [**4**]. We give the parameters of all pairs of strongly regular graphs whose tensor product admits primitive strongly regular fusions, and list the resulting feasible srg parameters. An imprimitive srg is a complete n-partite graph or its complement. We include these as possible factors in the tensor products, but require primitivity for successful fusions.

In Section 2 we give definitions and background material. Section 3 is a discussion of rank 3 srg's, followed by an example. In Section 4 we outline the method of working with intersection matrices and their eigenvalues. Section 5 is a summary of results, with remarks on some of the interesting cases that arise.

2. Preliminaries.

Definition. A *strongly regular graph* is a regular graph, neither complete nor null, with the property that the number of vertices

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