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CONSTRUCTIONS AND 3-DEFORMATIONS OF 2-POLYHEDRA AND GROUP PRESENTATIONS

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ABSTRACT. In this paper we shall study the Andrews-Curtis conjecture (AC) and its relation with some other interesting and important conjectures in low dimensional topology through special polyhedra and special presentations. An abelian monoid is constructed in the set of equivalence classes of mutually 3-deformable, contractible special polyhedra. It is shown that this abelian monoid is trivial if and only if (AC) is true. Some properties of this monoid are also discussed. Via the generalized Nielsen operations, we see the connection between special polyhedra and group presentations; hence, we derive another version of (AC). Then we prove that some cases of this version of (AC) are true.

1. Introduction. In 1964, E.C. Zeeman made the conjecture (Z) [**16**]: Every compact contractible 2-polyhedron is 1-collapsible. He also showed that (Z) implies the 3-dimensional Poincaré conjecture (3D-P). In 1965 Andrews and Curtis made their conjecture (AC) [**2**]: Every balanced, finite presentation of the trivial group can be reduced to the empty presentation by the generalized Nielsen operations. Later in 1975, P. Wright showed an equivalent formulation of (AC) [**13**]: Every compact contractible 2-polyhedron 3-deforms to a point. Because of this equivalent geometric formulation of (AC), it is easy to see that (Z) implies (AC). To understand these conjectures better and find a way to prove or disprove them, mathematicians are looking for their relationships. In 1983, Gillman and Rolfsen showed [**5**] that (Z) for thickened special polyhedra is equivalent to (3D-P). In 1987, S.V. Matveev claimed [**9**] that (Z) for unthickened special polyhedra is equivalent to (AC). In Section 2 we give most of the definitions for the future discussion in this paper. In Section 3 we construct $(\mathcal{M}, +)$ and show that it is an abelian monoid. Then we discuss the significance of $(\mathcal{M}, +)$ and the analogy between $(\mathcal{M}, +)$ and M.M. Cohen's geometric construction of the Whitehead group $\mathrm{Wh}\,(L)$ for a CW complex L. This analogy may give us some hint for calculating $(\mathcal{M}, +)$. In Section

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