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VELOCITY DEPENDENT BOUNDARY CONDITIONS FOR THE DISPLACEMENT IN A ONE DIMENSIONAL VISCOELASTIC MATERIAL

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Introduction. Conservation of mass and momentum in one dimension may be written as

$$\rho_0(X) = \rho(t, X)\frac{\partial y}{\partial X}, \tag{0.1a}$$

$$\rho_0(X)\frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial X}, \tag{0.1b}$$

where v is the velocity, $\rho_0(X)$ is the initial density, X is the material coordinate, y the spatial coordinate, and σ is the stress. These equations do not depend on the material under consideration. It is convenient to introduce the Lagrange mass variable [**17**], x defined by

$$x = \int_0^X \rho_0(z)\,dz.$$

Then writing (0.1b) in terms of the Lagrange mass variable,

$$\frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x}. \tag{0.2}$$

It is assumed in this paper that $\sigma = -(P + q)$ where P is pressure and q is the part of the stress due to viscosity. We also assume internal energy is constant so that it makes sense to let

$$P = P(V)$$

where V is the specific volume, $V = \rho^{-1}$ for ρ the density.

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