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ON A SUBCLASS OF STARLIKE FUNCTIONS

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ABSTRACT. Let $R(\beta)$ denote the class of functions $f(z) = z + a_2 z^2 + \cdots$ which are analytic in the unit disc $D = \{z : |z| < 1\}$ and satisfy the condition $\operatorname{Re}(f'(z) + zf''(z)) > \beta$, $\beta < 1$, for $z \in D$. We find β so that $R(\beta)$ is a subclass of S^*, the class consisting of univalent starlike functions in D.

1. Introduction. Let A denote the class of functions f which are analytic in the unit disc $D = \{z : |z| < 1\}$ and normalized so that $f(0) = f'(0) - 1 = 0$. Let S be the subclass of A consisting of univalent functions and let K and S^* denote the usual subclasses of S whose members are convex and starlike, respectively. For $\beta < 1$, let

$$R(\beta) = \{f \in A : \operatorname{Re}(f'(z) + zf''(z)) > \beta,\ z \in D\}.$$

The class $R(0)$ will simply be denoted by R. Chichra [**1**] proved that if $f \in R$, then for $z \in D$, $\operatorname{Re} f'(z) > 0$, and hence $R \subset S$. Singh and Singh [**7**] showed that $f \in R$ would imply $f \in S^*$ and Krzyz [**2**] gave an example to show that R is not a subset of K.

Let

$$\beta_S = \inf\{\beta : R(\beta) \subset S\},$$

and

$$\beta_{S^*} = \inf\{\beta : R(\beta) \subset S^*\}.$$

In a later paper, Singh and Singh [**8**] showed that $\beta_{S^*} \leq -1/4$. More recently, the estimate on β_{S^*} was further improved by Nunokawa and Thomas [**3**]. They proved that $R(\beta_0) \subset S^*$ if β_0 satisfies the equation

$$3\beta + (1 - \beta)(2 - \log(4/e))\log(4/e) = 0,$$

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