ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 3, Summer 1994

SYMMETRIES, TOPOLOGICAL DEGREE AND A THEOREM OF Z.Q. WANG

PATRICK J. RABIER

ABSTRACT. The primary purpose of this paper is to give a technically simpler proof of a formula for the Leray-Schauder degree of compact perturbations of the identity covariant under the smooth action of a compact Lie group G, recently proposed by Z.Q. Wang in the Hilbert space setting. Doing so, we eliminate questions raised by a few gaps of variable importance in Wang's proof, we extend the validity of his result to arbitrary Banach space and arbitrary continuous linear actions, and we show that in all the cases when the formula is not both trivial and useless, it depends only upon the action of a finite group (the factor group $N(T)/T$ where T is any maximal torus of the identity component of G and $N(T)$ its normalizer in G) in some appropriate subspace (the fixed point space of T). This is important regarding the practical value of the formula.

1. Introduction. In the recent paper [**20**] devoted to the calculation of the Leray-Schauder degree in presence of symmetries, Wang gives the following result:

Theorem 1.1. *Let X be a real Hilbert space, and let G be a compact Lie group acting in X through a smooth orthogonal representation in $GL(X)$. On the other hand, let $\Omega \subset X$ be a bounded G-invariant subset and $f \in C^0(\overline{\Omega}; X)$ a G-covariant compact perturbation of the identity such that $0 \notin f(\partial\Omega)$. Then*

$$d(f, \Omega, 0) = d(f^G, \Omega^G, 0) \bmod \mathcal{I}^G, \tag{1.1}$$

where $\Omega^G = \Omega \cap X^G$, X^G is the fixed point space of G, $f^G = f_{|\overline{\Omega} \cap X^G}$, and $\mathcal{I}^G$ denotes the ideal of $\mathbf{Z}$ generated by the Euler-Poincaré characteristics $\chi(G/G_x)$, $x \in X \backslash X^G$, G_x being the isotropy subgroup of x.

Formula (1.1) generalizes all previously known results, mostly devoted to finite $\wp$-groups or tori, regarding the calculation of $d(f, \Omega, 0)$ when

Received by the editors on September 3, 1992.