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# CARLESON'S INEQUALITY AND QUASICONFORMAL MAPPINGS

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**1.1. Introduction.** In his work on the interpolation of analytic functions Carleson characterized certain measures on the unit disc by means of $L^p$-integral inequalities for functions in $H^p$. Duren extended Carleson's theorem to exponents $0 < p \leq q < \infty$. We prove here analogues of these results for quasiconformal mappings in $\mathbf{R}^n$.

We denote the unit ball in $n$-dimensional Euclidean space, $R^n$, by $B^n$, and $S^{n-1}$ denotes its boundary. The open ball centered at $x \in R^n$ of radius $r$ is denoted $B(x, r)$. We assume throughout that $\mu$ is a positive measure on $B^n$. We call $\mu$ a $t$-Carleson measure, $0 < t < \infty$, if there exists a constant $N(\mu)$ such that

$$\mu(B(s,r) \cap B^n) \leq N(\mu) r^{t(n-1)} \tag{1.2}$$

for all $s \in S^{n-1}$ and all $0 < r < \infty$. When $n = 2$ and $t = 1$, this is Carleson's original definition [**3**].

The main result of this paper, Theorem 1.3, is a quasiconformal analogue of results of Carleson [**3**] and Duren [**4**] concerning analytic functions. To obtain this result, we use certain integral inequalities for the nontangential maximal function given in [**1**] and [**8**].

When $f : B^n \to R^m$ is measurable and $0 < p < \infty$, we write

$$||f||_{H^p} = \limsup_{r \to 1} \left( \int_{S^{n-1}} |f(rs)|^p \, d\sigma(s) \right)^{1/p}$$

where $d\sigma$ is the surface area measure on $S^{n-1}$.

We use here the usual definition of a $K$-quasiconformal mapping as defined in [**7**].

**Theorem 1.3.** *Suppose that* $0 < p \leq q < \infty$. *If* $t = q/p$ *and if*

$$\mu \text{ is a } t\text{-Carleson measure}, \tag{1.4}$$

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