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A SPLITTING CRITERION FOR A CLASS OF MIXED MODULES

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1. Introduction. This paper deals with mixed modules over discrete valuation domains. In particular, the results hold for local mixed abelian groups.

We study a class of mixed modules $\mathcal{H}$ with the property that the torsion submodule is a direct sum of cyclics and the quotient modulo the torsion is divisible of arbitrary rank. We show that the property of splitting of an arbitrary mixed module can be decided if a relatively small homomorphic image of a submodule of the mixed module splits in $\mathcal{H}$. To help formulate a necessary and sufficient splitting criterion for the modules in the class $\mathcal{H}$ we describe the modules by generators and relations. We then introduce the concept of a small relation array and show that mixed modules in $\mathcal{H}$ with this property split. The result of Baer and Fomin on the splitting of modules in $\mathcal{H}$ with bounded torsion submodules becomes an easy corollary. We conclude with two examples, one with a small relation array and the other a nonsmall relation array.

Let R denote a *discrete valuation domain*, i.e., a local principal ideal domain with prime p. All modules are always understood to be R-modules. A module G is said to *split* if its torsion submodule $\mathbf{t}G$ is a direct summand.

Recall that a *basic submodule* is a pure submodule which is a direct sum of cyclic modules and has divisible quotient. By [**1**, Section 32] every module G contains a basic submodule B. This basic submodule B is the direct sum of its torsion submodule $\mathbf{t}B$ and a free submodule F. If L is any free pure submodule of the module G, then by [**1**, Section 32, Exercise 7] there is a free pure submodule F containing L such that $G/(\mathbf{t}G \oplus F)$ is torsion-free divisible. Such a pure free module F with torsion-free divisible quotient $G/(\mathbf{t}G \oplus F)$ is called *relatively maximal pure free* in G. A pure free submodule is relatively maximal if and

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