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REES RINGS AND DERIVATIONS

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ABSTRACT. Let A be a ring, $\{I_n\}$ a filtration of ideals of A and $R = \oplus I_n T^n$ (contained in $A[T]$) the Rees ring associated with $\{I_n\}$. We study the derivations D of $A[T]$ such that $D(A) \subset A$ and $D(R) \subset R$.

Introduction. Let A be a noetherian ring and $\{I_n\}_{n\in\mathbf{Z}}$ a filtration of ideals of A. Let $R = \oplus_{n\geq 0} I_n T^n$ (respectively $R' = \oplus_{n\in\mathbf{Z}} I_n T^n$) be the Rees ring associated with $\{I_n\}$ for $n \geq 0$ (respectively for $n \in \mathbf{Z}$). One can remark that $R \subset A[T]$ and when $F = \{I^n\}$ (where I is an ideal of A) then R is the well-known "Rees algebra."

In this paper we first consider derivations D of the polynomial ring $A[T]$ such that $D(A) \subset A$, and we determine several conditions on $D(T)$ and $D(I_n)$ in order that $D(R) \subset R$ and $D(R') \subset R'$. In particular, we discuss five filtrations, namely $\{I^n\}$, $\{I^{(n)}\}$, $\{I^n : \langle J \rangle\}$, $\{(I^n)_\Delta\}$, $\{(I^n)_a\}$ (see definitions 1.4, 1.6, 1.8, 1.9).

In Section 2 we consider the Rees rings associated to the previous five filtrations. If $D \in \operatorname{Der}(A[T])$ is a derivation of one of these rings, we wonder on which of the others D is also a derivation. We give several implications and show some examples of implications which do not hold.

Further, if A is a noetherian domain containing a field of characteristic zero, for any filtration $\{I_n\}$ in A we show that each $D \in \operatorname{Der}(A[T])$ such that $D(R) \subset R$ is also a derivation of the Rees rings associated respectively to $\{\bar{I}_n\}$ and $\{(I_n)_a\}$ (where $\bar{I}_n$ (respectively, $(I_n)_a$) is the integral closure of I_n in $\bar{A}$ (respectively in A), see definition 1.9).

We recall that several properties of R have been studied in some cases. For example, when $F = \{I^{(n)}\}$ (I prime, i.e., R is the "symbolic Rees algebra"), many authors have studied when R is Noetherian, Gorenstein, Cohen-Macaulay (see [**1, 2, 3, 4**]). Further, in [**12**] there are some finiteness results related to certain filtrations.

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