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EXTREMAL DISKS AND COMPOSITION OPERATORS ON CONVEX DOMAINS IN $\mathbf{C}^n$

PETER R. MERCER

ABSTRACT. We obtain results about function spaces on strongly convex domains which are associated with the images of Kobayashi extremal disks. As an application we study composition operators on the Hardy function spaces of such domains.

0. Introduction. Let Ω be a smoothly bounded domain in $\mathbf{C}^n$, and let $M \subset \Omega$ be a complex submanifold which intersects $\partial\Omega$ transversally. It is natural to study function spaces on M in conjunction with function spaces on Ω. If $\Omega = B$, the unit ball, and M is a linear subspace, then a theorem of W. Rudin (Theorem 1.2 below) shows that the Hardy spaces $H^p(B)$ enjoy a simple relationship with the weighted Bergman spaces on M. A. Cumenge studied this situation in much greater generality in [**7**].

We focus on the situation where M is the image of a Kobayashi extremal disk. We generalize Rudin's result and make precise some of Cumenge's results for this special case. This work appears in Section 2 of this paper.

In Section 3 we apply these ideas, along with some work of M. Abate, to initiate the study of composition operators associated with Ω: Let $\Phi : \Omega \to \Omega$ be holomorphic. The composition operator T_Φ induced by Φ is defined by $T_\Phi(f) = f \circ \Phi$ (f is a holomorphic function on Ω). For $\Omega = B$ and $n \geq 2$, T_Φ is not in general a bounded operator on $H^p(B)$ [**5, 6**]. For Ω strongly convex we obtain necessary conditions on Φ for T_Φ to be a compact operator on $H^p(\Omega)$, thus generalizing some results of B. MacCluer for which $\Omega = B$ [**14**].

In Section 1 we fix some notation and isolate some results (mainly due to L. Lempert) which will be of subsequent use.

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