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A RESTRICTION-EXTENSION PROPERTY FOR OPERATORS ON BANACH SPACES

VANIA MASCIONI

ABSTRACT. We study a natural property of operators between Banach spaces which is shared by the class of operators factoring through a Hilbert space. This leads in particular to an operator theoretic version of the Lindenstrauss-Tzafriri characterization of Hilbertian spaces. Also, we point out connections to a classical result of Johnson-König-Maurey-Retherford.

Lindenstrauss and Tzafriri [**6**] proved the celebrated characterization: *a Banach space is isomorphic to a Hilbert space if and only if there exists a constant* $C > 0$ *such that, whenever* E *is a finite dimensional subspace of* X*, we can find an operator* R *on* X *such that* $R|_E \equiv \mathrm{id}_E$*,* $RX = E$ *and* $||R|| \leq C$. It is clear that the operator R is a projection. This reformulation of the Lindenstrauss-Tzafriri result suggests what seems to be a natural operator theoretic analogon of a space in which every subspace is complemented:

Definition. Let $T : X \to Y$ be an operator between Banach spaces X and Y. T has property (H_∞) if for every closed subspace $Z \subset X$ the restriction $T|_Z$ admits a *bounded extension* $R : X \to Y$ with range $RX \subset \overline{TZ}$.

With only minor changes, the Davis, Dean, Singer argument of [**1**] applies to show that, given T with (H_∞), the "extensions of the restrictions" R corresponding to finite dimensional subspaces Z of X can be chosen to be uniformly bounded. In terms of the next definition this means that *if* T *has* (H_∞)*, then* T *has* (H):

Definition. We say that $T : X \to Y$ has *property* (H) if there exists a positive constant C such that whenever E is a finite dimensional

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