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CHARACTERISTIC 4 WITT RINGS THAT ARE THE PRODUCT OF GROUP RINGS

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ABSTRACT. Necessary and sufficient conditions for a Witt ring of characteristic four to be the Witt product of group rings are presented. The proof is similar to a result of M. Marshall for Witt rings of characteristic two.

The definition of a Witt ring is that of [**5**]. Following [**3**], G will denote the group of units associated with the Witt ring R, and $q : G \times G \to R$ will denote the quaternionic map. For $x \in G$, $D\langle 1, x\rangle$ denotes the value group of the Pfister form $\langle 1, x\rangle$; if one takes the quaternionic map as the primitive concept one may define $D\langle 1, x\rangle = \{y \in G \mid q(y, -x) = 0\}$. For $S \subseteq G$, $\mathrm{gr}\,(S)$ denotes the group generated by the set S.

Motivation for the main result comes from [**4**], where Marshall proves:

Theorem 1. *Suppose R is a Witt ring of characteristic two. Then, in the category of Witt rings, R is a product of n group rings if and only if there exists an element $a \neq 1$ in G satisfying*:

(i) $\mathrm{rad}\,(a) = D\langle 1, a\rangle$ *has* 2^n *elements and*

(ii) $D\langle 1, b\rangle D\langle 1, ab\rangle = G$ *holds for all* $b \in \mathrm{rad}\,(a)$.

The main result of this paper is an analogy of Theorem 1 for Witt rings of characteristic four. Making the correct analogy depends on the simple observation that $1 = -1$ for rings of characteristic two, and so the element a of Theorem 1 is not equal to -1; but -1 can serve as this special element when $\mathrm{char}\, R = 4$.

The main result:

Theorem 2. *Suppose R is a nondegenerate Witt ring of characteristic 4. Then in the category of Witt rings, R is a product of n group rings if and only if*:

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