ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 4, Fall 1994

ON THE BLOW-UP OF SOLUTIONS TO SOME SEMILINEAR AND QUASILINEAR REACTION-DIFFUSION SYSTEMS

S.L. HOLLIS AND J.J. MORGAN

ABSTRACT. After a brief discussion of known global well-posedness results for semilinear systems, we introduce a class of quasilinear systems and obtain spatially local estimates which allow us to prove that if one component of the system blows up in finite time at a point x^* in space then at least one other component must also blow up at the same point. For a broad class of systems modelling one-step reversible chemical reactions, we show that blow-up in one component implies blow-up in *all* components at the same point in space and time.

1. Introduction. Considerable research has been done in the last decade on the problem of global well-posedness of semilinear parabolic systems of partial differential equations; i.e., reaction-diffusion systems. See, e.g., [**1**–**7**, **9**–**13**]. A system is said to be globally well-posed if classical solutions continue for all time $t > 0$ given any nonnegative L^∞ initial data. Perhaps the greatest source of interesting problems in this area is the modelling of multi-species chemical reactions. For example, let us consider the following, seemingly simple, reversible reaction in which sulphur dioxide reacts with oxygen to form sulphur trioxide:

$$2SO_2 + O_2 \rightleftharpoons 2SO_3. \tag{1.1}$$

If we set $A = [SO_2]$, $B = [O_2]$, and $C = [SO_3]$, then this reaction, assuming mass action kinetics, may be modelled by the reaction-diffusion system:

$$\begin{aligned} A_t - d_1\Delta A &= 2(k_r C^2 - k_f A^2 B) \\ B_t - d_2\Delta B &= k_r C^2 - k_f A^2 B \\ C_t - d_3\Delta C &= 2(k_f^2 B - k_r C^2) \end{aligned} \tag{1.2}$$

Received by the editors on February 8, 1993, and in revised form on April 9, 1993.