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DIMENSION c OF ORBITS AND CONTINUITY OF TRANSLATION FOR SEMIGROUPS

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ABSTRACT. Let S be a semi-topological semigroup and Φ a Banach space on which S acts as a semigroup of linear isometries. Let $\bar{x}\mu$ denote the effect of $x \in S$ on $\mu \in \Phi$. In many cases, we have that either the orbit $\{\bar{x}\mu : x \in S\}$ is nonseparable, or $x \mapsto \bar{x}\mu$ is norm-continuous. We investigate when "nonseparable" can be replaced with "spans a closed subspace of topological dimension at least c." We thus extend results known for the case that S is a group. We given examples and related results.

0. Introduction. Let S be a locally compact group, Φ a Banach space, and $x \mapsto \bar{x}\mu$, $\mu \in \Phi$, $x \in S$ a representation of S that is lower semicontinuous. Then for each $\mu \in \Phi$, $x \mapsto \bar{x}\mu$ is either continuous, or $\{\bar{x}\mu : x \in U\} = \mathcal{O}(\mu, U)$ is nonseparable for each open U; in fact, $\mathcal{O}(\mu, U)$ spans a subspace of topological dimension at least c [**1, 3**].

A semigroup with a topology S is a "semi-topological" semigroup if the semigroup operation is continuous in each variable separately; it is a "topological" semigroup if the semigroup operation is continuous in both variables simultaneously. All semigroup topologies will be assumed to be locally compact.

When S is a topological semigroup, $x \mapsto \bar{x}\mu$ can be discontinuous and $\mathcal{O}(\mu, S)$ separable: we give examples. In this paper we identify conditions under which $\mathcal{O}(\mu, S)$ is nonseparable when $x \mapsto \bar{x}\mu$ is not continuous, apply them to the group case, and provide examples that illustrate the limits of what is possible.

Most of our examples are inspired by what occurs in the case of $\Phi = M(S)$, the space of all regular bounded Borel measures on the

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