ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 4, Fall 1994

A QUASI INNER PRODUCT APPROACH FOR CONSTRUCTING SOLUTION REPRESENTATIONS OF CAUCHY PROBLEMS

L.R. BRAGG

ABSTRACT. Integral representations for a wide class of Cauchy problems are developed by employing the method of quasi inner products. This approach reduces the construction of solutions to translations followed by integrations. Bounds on solutions corresponding to polynomial data are obtained and these permit proving series representation theorems in terms of special polynomial and other sets. Among the problems considered are ones associated with the Yukawa, the Helmholtz, the EPD and the GASPT equations. Some nonstandard and higher order equations are also considered.

1. Introduction. Function theoretic methods have played a significant role in the development of solutions of partial differential equations and in deducing their properties. These methods have a long and rich history and ongoing refinements continue to lead to new and important results for initial and boundary value problems. Major innovators in this subject include E.T. Whittaker [**23, 24**], S. Bergman [**2**] and I. Vekua [**20**]. The list of authors who have made contributions in this area is indeed extensive and the reader is further referred to the treatises of R.P. Gilbert [**17**] and K.W. Bauer and S. Ruscheweyh [**1**] for a broad coverage of the subject and for their extensive bibliographies.

At the heart of most of these approaches for solving partial differential equations is the Cauchy integral formula and various modifications of it. In this paper we employ one such version which permits the construction of integral and series representations for a wide class of Cauchy problems that involve, for example, the wave equation, the Laplace equation, the Euler-Poisson-Darboux (EPD) equation, the equation of generalized axially symmetric potential theory (GASPT), the Yukawa

Received by the editors on March 15, 1993, and in revised form on July 13, 1993.
AMS MOS *Subject Classification.* Primary 35A20, Secondary 35L30, 35J05.
Key words and phrases. Quasi inner product, Cauchy problems, translation operators, expansion sets, Yukawa equation, Helmholtz equation, Euler-Poisson-Darboux equation.