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SPATIALLY NONTEMPERATE PSEUDODIFFERENTIAL OPERATORS, SPHERE EXTENSIONS AND FREDHOLM THEORY

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0. Introduction. There are two main approaches to the global study of pseudodifferential operators, henceforth abbreviated ΨDO's, on a noncompact manifold which, in what follows, will always be the Euclidean n-space $\mathbf{R}^n$. In the first (and most commonly used) approach, the "calculus method," one starts with certain classes of "symbols" having suitable growth conditions with respect to appropriate weight functions, and one then assigns a ΨDO to each symbol via the Fourier inversion formula. The detailed analysis of this method is given, e.g., by L. Hörmander (cf. [**5**, vol. III, and its bibliography]). Going in the opposite direction, the second approach, the "Gelfand theory method," begins by constructing certain "comparison" C^*-algebras of ΨDO's based on suitable Schrödinger-type operators, and then uses Gelfand theory to attach a "symbol" to each ΨDO. This method is developed in detail by H.O. Cordes (cf. [3] and the references therein). Our goal in this paper is to look at the relation between the above methods, by following both of them in a rather general situation. In Section 1 we follow the "calculus method," introducing a class of symbols, not necessarily temperate in the space variables, with weight function

$$h(x,\xi) = (q(x) + |\xi|^2)^{1/2} \tag{0.1}$$

where $q(x)$ is a smooth (i.e., $C^\infty(\mathbf{R}^n)$) function satisfying

$$q(x) \geq 1, \quad \forall\, x \in \mathbf{R}^n, \qquad \lim_{|x|\to\infty} q(x) = \infty, \tag{0.2}$$

and

$$|\partial_x q(x)| = o(q(x)), \quad \text{as } |x| \to \infty, \tag{0.3}$$

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