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QUALITATIVE ANALYSIS OF NONLINEAR SYSTEMS BY THE METHOD OF MATRIX LYAPUNOV FUNCTIONS

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ABSTRACT. This paper surveys applications of the method of matrix Lyapunov functions in the investigation of stability, asymptotic stability and instability of systems modelling real phenomena in engineering and technology. As a framework of the stability analysis systems of ordinary differential equations (ODE) under structural perturbations, the concept of matrix Lyapunov function is used.

0. What is a matrix Lyapunov function? As is well known, an auxiliary function with properties similar to those of a norm is an essential tool in Lyapunov's direct method for the qualitative theory of differential equations. At present, scalar Lyapunov functions are widely used for the solution of various problems in engineering, technology, mathematical biology, economics, etc. In the early 1970s, there appeared techniques utilizing several auxiliary functions that were actually components of a vector Lyapunov function. The idea was extensively developed when the dynamical properties of solutions of so-called "large scale systems" were investigated.

Further development of ideas by Lyapunov and Poincare resulted in the concept of a "matrix auxiliary function." Our subsequent presentation deals with this concept.

0.1. Let $(\mathbf{R}^n, ||\cdot||)$ be a real Euclidean normed space. We denote by $B(\rho) = \{x : ||x|| < \rho\}$ an open ball with radius ρ and center at the origin, and $\mathcal{D} = \mathcal{T}_0 \times B(\rho)$ is a direct Descart product, where $\mathcal{T}_0 = \{t : t_0 \leq t < \infty\}$ and $t_0 \in \mathcal{T}_\tau \subseteq \mathbf{R}$, where $\mathcal{T}_\tau = \{t : \tau \leq t < +\infty\}$,

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