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ON BIFURCATION AND EXISTENCE OF POSITIVE SOLUTIONS FOR A CERTAIN p-LAPLACIAN SYSTEM

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1. Introduction. In this paper we study bifurcation of positive solutions for an elliptic system of the form

$$\begin{cases} -\Delta_p u_i + g_i(x, u_1, u_2) = \lambda_i |u_i|^{p-2} u_i & \text{in } \Omega \\ u_i = 0 & \text{on } \partial\Omega \end{cases} \qquad i = 1, 2 \tag{1.1}$$

on a smooth bounded domain Ω in $\mathbf{R}^N$, where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian with $p > 1$. We will prove that under appropriate conditions on g_i, (1.1) has a continuum of positive solutions bifurcating from the trivial solution. In particular, it follows from our main result (Theorem 3.1) that the following competitive system

$$\begin{aligned} -\Delta_p u_1 &= |u_1|^{p-2} u_1 (\lambda_1 - a_{11} u_1 - a_{12} u_2) \quad \text{in } \Omega \\ -\Delta_p u_2 &= |u_2|^{p-2} u_2 (\lambda_2 - a_{21} u_1 - a_{22} u_2) \quad \text{in } \Omega \\ u_i &= 0, \qquad i = 1, 2 \quad \text{on } \partial\Omega \end{aligned} \tag{1.2}$$

admits positive solutions (u_1, u_2), with $u_i > 0$, for some positive λ_i and a_{ij}, $i, j = 1, 2$.

When $p = 2$, the p-Laplacian becomes the usual Laplacian and system (1.1) has been studied extensively. We refer to the work of Cantrell [**5**] and the reference therein. In the case when $p \neq 2$, Δ_p appears in numerous situations. For example, in the context of reaction-diffusions, Murray [**16**] suggested using diffusion of the form $\Delta_p u$ in the study of diffusion-kinetic enzymes problems. We mention [**7**] and [**4**] for other references. Recently, systems associated with the p-Laplacian have commanded growing interest. Fleckinger et al. [**11, 12**] studied the

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