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ON AN L^1-FORCED AUTONOMOUS DUFFING'S EQUATION WITH PERIODIC BOUNDARY CONDITIONS IN THE PRESENCE OF DAMPING

CHAITAN P. GUPTA

ABSTRACT. Let $g : \mathbf{R} \to \mathbf{R}$ be a continuous function, $e : [0,1] \to \mathbf{R}$ be a function in $L^1[0,1]$ and $c \in \mathbf{R}$, $c \neq 0$, be given. Suppose that $\alpha \in \mathbf{R}$, $1 \leq \alpha < 2$ be such that $\lim_{|u|\to\infty} |\frac{g(u)}{u^\alpha}| < \infty$, and let $g_- = \limsup_{\alpha\to-\infty} g(u), g_+ = \liminf_{u\to\infty} g(u)$ so that $-\infty \leq g_- < g_+ \leq \infty$. Then if $g_- < \int_0^1 e(x)\,dx < g_+$, the boundary value problem $u'' + cu' + g(u) = e$, $u(0) = u(1)$, $u'(0) = u'(1)$ has at least one solution. It is also proved that if g is increasing in $\mathbf{R}$ (not necessarily strictly) and g is Lipschitz-continuous with Lipschitz constant k, such that $k < 4\pi^2 + c^2$ then the set of solutions of $u'' + cu' + g(u) = e, u(0) = u(1)$, $u'(0) = u'(1)$ is a non-empty, compact, connected and acyclic set.

1. Introduction. Let $g : \mathbf{R} \to \mathbf{R}$ be a continuous function, $e : [0,1] \to \mathbf{R}$ and $c \in \mathbf{R}$, $c \neq 0$, be given. This paper is devoted to the study of the forced autonomous Duffing's equation

$$
\begin{gathered}
u'' + cu' + g(u) = e(x), \qquad 0 < x < 1 \\
u(0) = u(1), \qquad u'(0) = u'(1).
\end{gathered} \tag{1.1}
$$

This equation was studied by the author in [**2**], when $e(x) \in L^2[0,1]$. It was proved in [**2**] that if $g_- = \limsup_{u\to-\infty} g(u)$, $g_+ = \limsup_{u\to\infty} g(u)$ and $-\infty \leq g_- < \int_0^1 e(x)\,dx < g_+ \leq \infty$, then the equation (1.1) has at least one solution. The motivation to study equation (1.1) came from the observation that, if $c \neq 0$, the linear boundary value problem

$$
\begin{gathered}
u'' + cu' = \lambda u, \qquad 0 < x < 1, \\
u(0) = u(1), \qquad u'(0) = u'(1),
\end{gathered} \tag{1.2}
$$

has $\lambda = 0$ as its only eigenvalue.

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