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UNRAMIFIED QUADRATIC EXTENSIONS OF A QUADRATIC FIELD

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ABSTRACT. Given a quadratic field K, we determine the number of quadratic extensions of K, which are unramified at all finite primes.

Let Q denote the field of rational numbers. Let K be a quadratic extension of Q, and let L be a quadratic extension of K. We show (Theorem 1) that if L/K is unramified at all finite primes then L is a bicyclic extension of Q, that is, $\mathrm{Gal}\,(L/Q) \simeq Z_2 \times Z_2$. Then in Theorem 2 we give the precise form of those bicyclic extensions L such that L/K is unramified at all finite primes. Theorem 2 then enables us to determine for a given quadratic field K the number of unramified quadratic extensions of K (Theorem 3).

Theorem 1. *Let K be a quadratic extension of Q. Let L be a quadratic extension of K. If L/K is unramified at all finite primes, then L is a bicyclic extension of Q.*

Proof. As K is a quadratic extension of Q, we have $K = Q(\sqrt{c})$, where c is a square-free integer not equal to 1. As L is a quadratic extension of K, there exists a nonsquare integer μ of K such that $L = K(\sqrt{\mu})$. We set $\mu O_K = RS^2$, where R and S are integral ideals of the ring O_K of integers of K with R square-free. It was shown in [**5**, Theorem 1] that the relative discriminant of L over K is given by $d(L/K) = RT^2$, for some integral ideal T of O_K. As L/K is unramified at all finite primes, we have $R = T = O_K$, and thus $\mu O_K = S^2$.

Let $P_1, \dots, P_t$ be the distinct prime ideals of O_K which divide $d(K) = c$ or $4c$. It is well known (see, for example, [**1**, p. 249]) that the class

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