ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 2, Spring 1995

NONLINEAR FOURTH-ORDER TWO-POINT BOUNDARY VALUE PROBLEMS

FELIX SADYRBAEV

ABSTRACT. Existence results formulated in terms of upper and lower functions and Nagumo type conditions are presented for fourth order equations and systems of two second order equations with nonlinear boundary constraints.

1. Introduction. This paper deals with the solvability of boundary value problems associated with fourth order differential equations

$$x^{(4)} = f(t, x, x', x'', x''') \tag{1}$$

as well as with systems of the form

$$\begin{aligned} x'' &= f_1(t, x, y, x', y'), \\ y'' &= f_2(t, x, y, x', y'). \end{aligned} \tag{2}$$

The results are inspired by the ones given by Bebernes and Fraker [**3**] for the boundary value problem

$$x'' = f(t, x, x'), \tag{3}$$

$$(x(0), x'(0)) \in S_1, \qquad (x(1), x'(1)) \in S_2, \tag{4}$$

where the function f is supposed to be continuous on $[0, 1] \times \mathbf{R}^2$ and the boundary sets S_1 and S_2 are subsets of $\mathbf{R}^2$. The main idea in [**3**] is to investigate the dependence of S_1 and S_2 on the a priori bounds in order that the BVP (3), (4) have a solution.

The standard set of conditions ensuring a priori bounds is the following.

(A) *Lower and upper functions.* There exist functions α and β satisfying on $[0, 1]$ the inequalities $\alpha \leq \beta$,

$$\alpha'' \geq f(t, \alpha, \alpha'), \qquad \beta'' \leq f(t, \beta, \beta'). \tag{5}$$

Received by the editors on May 28, 1991, and in revised form on March 30, 1993.
Key words. Nonlinear boundary value problems, a priori bounds, upper and lower functions.
AMS 1991 *Mathematics Subject Classification.* 34B15.