ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 2, Spring 1995

GENERALIZED FEYNMAN INTEGRALS: THE $\mathcal{L}(L_2, L_2)$ THEORY

CHULL PARK AND DAVID SKOUG

ABSTRACT. In this paper we develop an $\mathcal{L}(L_2(\mathbf{R}), L_2(\mathbf{R}))$ theory for the Feynman integral of functionals of general stochastic processes.

1. Introduction. In [**1**], Cameron and Storvick introduced a very general analytic operator-valued function space *Feynman integral*, $J_q^{an}(F)$, which mapped an $L_2(\mathbf{R})$ function ψ into an $L_2(\mathbf{R})$ function $(J_q^{an}(F)\psi)(\xi)$. Further work involving the $L_2 \to L_2$ theory, the $L_1 \to L_\infty$ theory and the $L_p \to L_{p'}$ theory, $1/p + 1/p' = 1$, includes [**2, 3, 11, 12, 13**].

In [**9**], Chung and Skoug introduced the concept of a conditional Feynman integral using Yeh's definition of conditional Wiener integrals [**20**]. In [**7**], Chung, Park and Skoug expressed the Feynman integral $J_q^{an}(F) \in \mathcal{L}(L_1(\mathbf{R}), L_\infty(\mathbf{R}))$ in terms of conditional Feynman integrals.

In various Feynman integration theories, the integrand F of the Feynman integral is a functional of the standard Wiener (i.e., Brownian) process. In [**8**], Chung, Park and Skoug defined a Feynman integral for functionals of general stochastic processes. They then used the theory of the conditional Feynman integral to develop an $\mathcal{L}(K(\mathbf{R}), L_\infty(\mathbf{R}))$ theory where

$$K(\mathbf{R}) = \{\psi_1 + \psi_2 : \psi_1 \in L_1(\mathbf{R}) \text{ and } \psi_2 \in \hat{M}(\mathbf{R})\},$$

and where $\hat{M}(\mathbf{R})$ is the space of Fourier transforms of measures from $M(\mathbf{R})$, the space of $\mathbf{C}$-valued countably additive Borel measures on $\mathbf{R}$.

In this paper we develop an $\mathcal{L}(L_2(\mathbf{R}), L_2(\mathbf{R}))$ theory for the operator-valued Feynman integral of functionals of general stochastic processes. The $L_2 \to L_2$ theory is more relevant in quantum mechanics and other applications than the $L_1 \to L_\infty$ or the $K(\mathbf{R}) \to L_\infty(\mathbf{R})$ theory.

Received by the editors on August 5, 1992, and in revised form on September 27, 1993.