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WEAK COMPACTNESS IN SPACES OF DIFFERENTIABLE MAPPINGS

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ABSTRACT. We characterize the weakly compact subsets (and thereby the weak convergence) in several spaces of k-times continuously differentiable mappings between real Banach spaces. As an application, we give characterizations of the Dunford-Pettis (DP) property of a Banach space F in terms of the weak sequential continuity of the composition map $(f, g) \mapsto g \circ f$, where $f : E \to F$ is a differentiable mapping and $g : F \to G$ is a linear operator. We also prove that F has the DP property if and only if whenever $(x_n) \subset F$ is weakly null and (P_n) is a weakly null sequence of polynomials from F to another space G, then $(P_n(x_n))$ converges to 0 in the weak topology of G. Finally, we derive a new proof of the fact that any weakly compact homomorphism between algebras of differentiable functions is induced by a constant mapping.

1. Introduction. Kalton [**11**] characterized the weakly compact subsets of the space $K(E, F)$ of compact operators between Banach spaces E and F. These results were extended in [**4, 5**] to the case of compact operators between locally convex spaces.

In this paper we characterize the weakly compact subsets (and thereby the weak convergence) in several spaces of k-times continuously Fréchet differentiable mappings between real Banach spaces E and F: $C^k_{wu}(E, F)$ (definition below), $C^k_c(E, F)$ and $C^k(E, F)$ (definitions in Section 5). As an application, we prove that F has the Dunford-Pettis property if and only if, for every pair of real Banach spaces E, G and integer k, the composition map:

$$T : C^k_{wu}(E, F) \times L(F, G) \to C^k_{wu}(E, G)$$

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