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A GEOMETRIC CHARACTERIZATION OF THE WEAK-RADON NIKODYM PROPERTY IN DUAL BANACH SPACES

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ABSTRACT. We give a geometric characterization of convex, weak*-compact subsets of a dual Banach space with the weak-Radon Nikodym property as those sets in which every closed, convex subset is the weak*-closed convex hull of its x^{**}-weak*-strongly exposed points for each element x^{**} of X^{**}.

1. Introduction. After the characterization by Musial [**9**] and Janicka [**8**] of dual Banach spaces with the weak-Radon Nikodym property (that is, the Radon-Nikodym property for the Pettis integral) as the spaces with predual not containing l_1, many characteristic properties for the weak*-compact subsets of such spaces were proved (see [**7, 12**]). Many of these properties localized to provide equivalent properties for weak*-compact subsets of dual spaces [**6, 10, 11, 13**].

A convex, weak*-compact subset K of a dual Banach space X^* has the weak-Radon Nikodym property (w-RNP) if and only if it is a Pettis set [**5, 13**] or equivalently if it is weakly fragmented [**5**] (K is weakly fragmented if for every nonempty, w^*-compact subset F of K, $\varepsilon > 0$ and $x^{**} \in X^{**}$ there exists a nonempty, relatively open subset U of (F, w^*) such that $O(x^{**}, U) < \varepsilon$). Also, characteristic properties of a convex, weakly fragmented set K are that the norm-closed convex hull of F is equal to the weak*-closed convex hull of F for every weak*-compact subset F of K and that every convex, weak*-compact subset L of K is equal to the norm-closed convex hull of its extreme points [**5, 7**].

In this paper (see Theorem 8) we give a geometric characterization of convex, weak*-compact, with the w-RNP subsets of a dual Banach space as those sets in which every weak*-compact, convex subset is the weak*-closed convex hull of its *x^{**}-weak*-strongly exposed points* for each element x^{**} of X^{**}. An extreme point x^* of K is an x^{**}-weak*-strongly exposed point of K for some x^{**} in X^{**} if there exists

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