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HIGHER ORDER UNIFORMLY GÂTEAUX DIFFERENTIABLE NORMS ON ORLICZ SPACES

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ABSTRACT. Equivalent α_M-times uniformly Gâteaux differentiable norms are constructed for large classes of Orlicz spaces $L_M(S, \Sigma, \mu)$. Especially, for the spaces $L_{2p-1}(0,1)$, $p \in \mathbf{N}$, equivalent $(2p-1)$-uniformly Gâteaux smooth norms are found.

1. Introduction. The existence of smooth bump functions on a Banach space is of some importance in many problems of the nonlinear analysis. At the end of the 1980s, several deep results of Deville [**2, 3**] showed that the existence of higher order differentiable bumps also has geometrical implications.

The problem of the best order of Fréchet differentiability of bump functions was solved for L_p-spaces in [**1, 12**] and for Orlicz sequence spaces in [**9, 10**]. Especially, it is shown [**1**] that in l_p, p odd, there is no p-times Fréchet differentiable bump and [**9**] that in $l_M, \alpha_M^0 \in \mathbf{N}$, there is no α_M^0-times Fréchet differentiable bump, excepting the case where α_M^0 is even and M is equivalent to $t^{\alpha_M^0}$ at 0.

On the other hand, in a Banach space, a norm of some order of smoothness generates a bump with the same order of smoothness and therefore every positive result on the existence of a smooth equivalent norm is transferred directly for bumps. In [**11**] equivalent p-times Gâteaux differentiable norms are found in L_p over σ-finite measure space, p odd. Our aim is to generalize and sharpen this result for Orlicz sequence spaces l_M (function spaces $L_M(0,1)$) with $\alpha_M^0(\alpha_M^\infty)$ a positive integer and M not equivalent to $t^{\alpha_M^0}(t^{\alpha_M^\infty})$ at $0(\infty)$.

2. Preliminaries. We begin with some notations and definitions. In what follows X and Y are Banach spaces, S_X and B_X the unit sphere

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